Intermediate Macroeconomics

Economic Growth

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Outline

- Growth Facts
- Production theory (Chapter 3)
    - What determines the economy's total output/income: how much do firm produce?
    - How total income is distributed?
- Growth theory
    - Growth theory I: Solow growth model (Chapter 8)
    - Growth theory II: Population and technological progress (Chapter 8, 9)

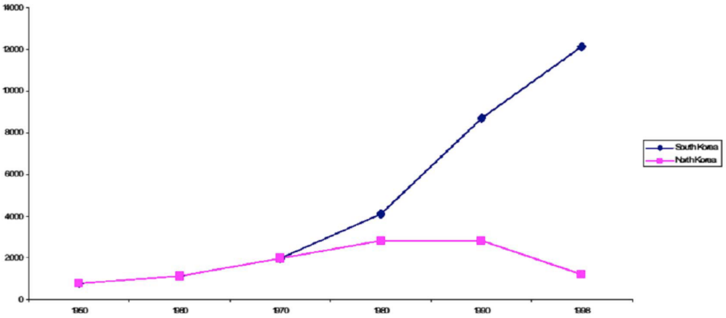
Growth Facts

Differences across countries

- The average American's income is around 5 bigger than the average Mexican's, around 14 times bigger than the average Indian's, and 35 times bigger than the average African's, all in PPP (comparable prices) terms.
- Life expectancy in rich countries is 77 years, 67 years in middle income countries, and 53 years in poor countries.
- Out of 6.4 billion people, 0.8 do not have access to enough food, 1 to safe drinking water, and 2.4 to sanitation.

Differences across countries

Figure 1: Evolution of Income Per Capita: North vs South Korea



Differences across time

- The average modern American is around 20 times richer than the average colonial American.
- An American worked 61 hours per week in 1870, today 34.
- Japanese boy born in 1880 had a life expectancy of 35 years, today 81 years.

A Question

Robert Lucas, 1988, p5

I do not see how one can look at figures like these without seeing them as representing possibilities. Is there some action a government could take that would lead the Indian economy to grow like Indonesia's? If so, what exactly? If not, what is it about the "nature of India" that makes it so?

Production Theory

Factors of Production

Firms utilize two inputs: **Capital** and **Labor**

- Capital K: tools, machines, and structures used in production
- Labor L: physical and mental efforts of workers

For the moment, we assume they are

- Fixed

$$K = \bar{K}$$
$$L = \bar{L}$$

- Fully utilized: e.g, no unemployment

Production Function

Firms utilize capital and labor to produce output, given the production function,

$$Y = F(K, L)$$

such that,

- Shows how much output Y the economy can produce from K and L units
- Reflects the economy's level of technology
- Exhibits **constant returns to scale.**

Return to scale: review

Initially, $Y_1 = F(K_1, L_1)$

Scale all inputs by the same factor z

- $z = 2$: firms double K and L

What happens to output $Y_2 = F(K_2, L_2)$?

- If **constant return to scale**, $Y_2 = zY_1$
- If **increasing return to scale**, $Y_2 > zY_1$
- If **decreasing return to scale**, $Y_2 < zY_1$

Returns to scale: Examples

1. $F(K, L) = \sqrt{KL}$
2. $F(K, L) = K^2 + L^2$
3. $F(K, L) = \frac{K^2}{L}$
4. $F(K, L) = K + L$

GDP: supply of goods and services

GDP: Gross Domestic Product

- measure the supply (we will re-visit GDP later)

Because we assumed factors of production (K and L) are fixed. The quantity of goods and services produced in economy (GDP) is given by,

$$Y = F(\bar{K}, \bar{L}) = \bar{Y}$$

Production generate income,

- question: how national income will be distributed among K and L?

Factor Market and the Distribution of National Income

- There are many ways to distribute national income
- In this chapter, we will focus on how national income is distributed between,
  - Workers (in the form of wages)
  - Capital Owners (capital rents)
- The allocation of national income between capital owners and workers is determined by factor prices, i.e. the prices per unit firms pay for the factors of production

Factor Prices

- Wage W : price of L
- Rental price of capital R: price of K

How are factor prices determined?

Factor prices are determined by supply and demand in factor markets

Factor markets,

- Workers supply labor, capital owners supply capitals
- Firms buy labor and capital, pay wage and rental rate
- We tentatively assume they're fixed (relax this assumption later)

Firms' object

Assumptions,

- Market is competitive
- There are many identical small firms. Since they are identical, we can aggregate them as one representative and competitive firm.

Firm uses a technology to transform inputs into outputs: production function

$$Y = F(K, L)$$

Sells output Y, hires workers at a wage W, and rents capital at a rate R

Firm chooses labor and capital to maximize profits π

$$\pi = P * Y - WL - RK$$

Since $Y = F(K, L)$,

$$\pi = \underbrace{P * F(K, L)}_{revenue} - \underbrace{WL}_{\text{labor costs}} - \underbrace{RK}_{\text{capital costs}}$$

Demand for Labor

By taking partial derivative respect to L on profit function $\frac{\partial \pi}{\partial L}$, and set equal to zero

$$\begin{aligned} P * \frac{\partial F(K,L)}{\partial L} - W &= 0 \\ P * \frac{\partial F(K,L)}{\partial L} &= W \\ \frac{\partial F(K,L)}{\partial L} &= \frac{W}{P} \end{aligned}$$

$\frac{\partial F(K,L)}{\partial L}$: **marginal product of labor, MPL**

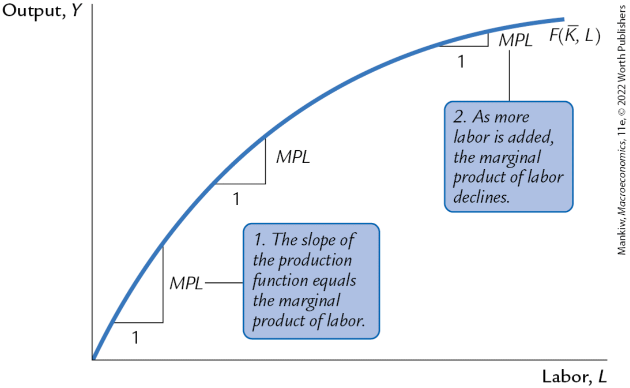
- The extra output the firm can produce using an additional unit of labor (holding other inputs fixed)
- Intuitively,

$$\text{MPL} = F(K, L+1) - F(K, L)$$

MPL

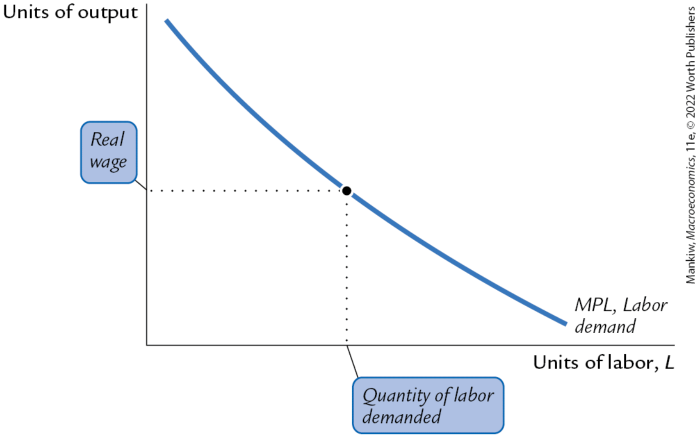
L	F(L,K)	F(L+1,K)	MPL
0	0	10	10
1	10	19	9
2	19	27	8
3	27	34	7
4	34	40	6
5	40	45	5
6	45	49	4
7	49	52	3
8	52	54	2
9	54	55	1
10	55		

Diminishing Marginal Returns



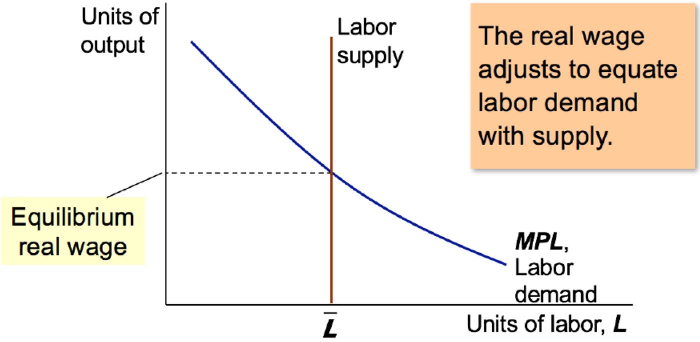
Hiring more labors will increase outputs, however each additional labor would produce less

Diminishing MPL



Equilibrium W

Since we assume that labor supply L is fixed



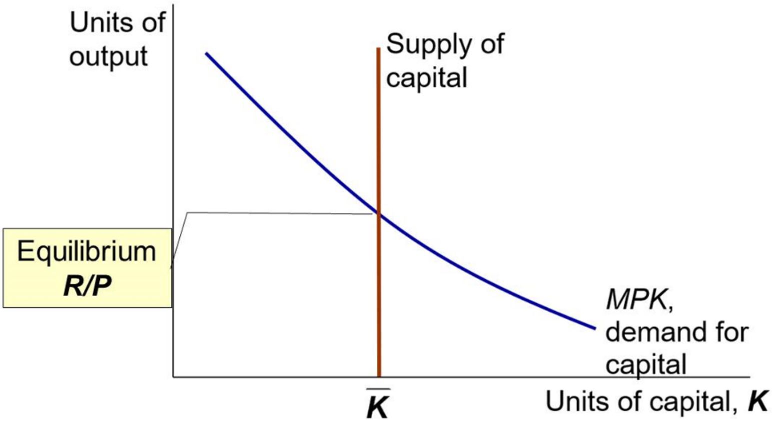
Demand for Capital

Same logic follows for capital

- Diminishing returns to capital
- Diminishing MPK
- Firms maximize profits by choosing K,

$$MPK = \frac{R}{P}$$

Equilibrium R



What if there are shocks?

Pandemics?

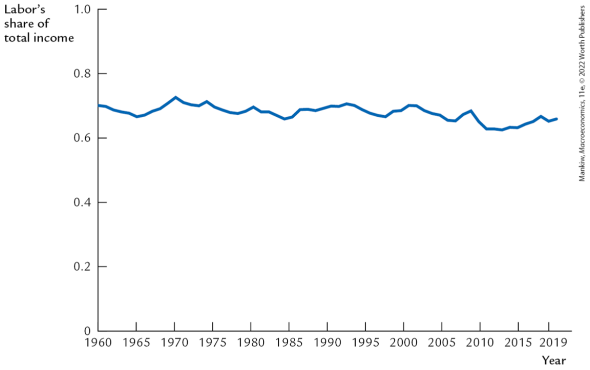
How Income Is Distributed to L and K

- Total (real) labor income $= \frac{W}{P} * \bar{L}$
- Total (real) capital income $= \frac{R}{P} * \bar{K}$

Therefore, total income in the economy,

$$\bar{Y} = \frac{W}{P}\bar{L} + \frac{R}{P}\bar{K}$$
$$\bar{Y} = \underbrace{MPL * \bar{L}}_{\text{labor income}} + \underbrace{MPK * \bar{K}}_{\text{capital income}}$$

How Much of National Income Goes to Labor Income?



- Labor share of income is approximately constant over time at around 0.7
- Thus, capital income share is 0.3

Cobb-Douglas Production Function

Motivated by the graph above, economists found a production function that had constant factor shares over time.

That is called the Cobb-Douglas production function:

$$Y = F(K, L) = AK^{\alpha}L^{1-\alpha} \quad \alpha \in [0, 1]$$

Where A represents the level of technological progress in the country and it is a constant in the model.

Cobb-Douglas Production Function

$$Y = F(K, L) = AK^{\alpha}L^{1-\alpha} \qquad \alpha \in [0, 1]$$

- $MPL = \frac{\partial Y}{\partial L} = AK^{\alpha}(1-\alpha)L^{-\alpha}$
- $MPK = \frac{\partial Y}{\partial K} = A\alpha K^{\alpha-1}L^{1-\alpha}$

Again,

- $MPL = W/P$
- $MPK = R/P$

MP in Cobb-Douglas Production Function

Filled by in-class notes

Back to our question

How much goes to workers and how much goes to capital owners?

A Numerical Example

$$Y = F(K, L) = AK^{\alpha}L^{1-\alpha} \quad \alpha \in [0, 1]$$

- Suppose $A = 2$, $\alpha = 0.5$, $K = 16$, $L = 100$
- How much is MPL, MPK, W, and R?
- If there is an earthquake destroy some K, so new $K = 9$. What will happen for MPL, MPK, W, and R?
- How much income goes to labor?

Solow Growth Model 1
- Capital Accumulation

Why Growth Matters

- Data on infant mortality rates:
  - 20% in the poorest 1/5 of all countries
  - 0.4% in the richest 1/5
- In Pakistan, 85% of people live on less than $2/day.
- One-fourth of the poorest countries have had famines during the past 3 decades.
- Poverty is associated with oppression of women and minorities.

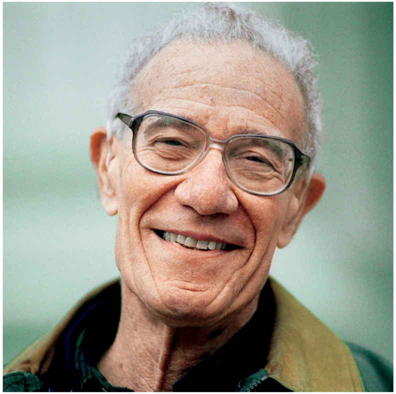
Economic growth raises living standards and reduces poverty Click Here

Re-visit Cobb-Douglas Production Function

$$Y = F(K, L) = AK^{\alpha}L^{1-\alpha} \quad \alpha \in [0, 1]$$

- Suppose A and L are constant or do not vary over time
- The only growth channel is capital K
- This is the simplest version of **Solow Growth Model**

Named after economist Solow, who won Nobel Prize in 1987



National Income and Production

Assume no government spending and trade, so GDP identity can be written as,

$$Y = \underbrace{C + I}_{expenditure}$$

- C: consumption
- I: investment

Also, we have production function,

$$Y = F(K, L) = AK^{\alpha}L^{1-\alpha} \quad \alpha \in [0, 1]$$

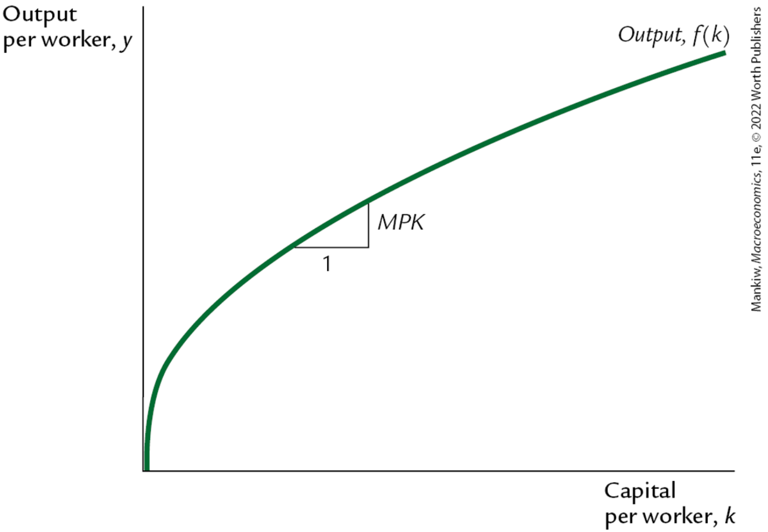
Supply of Goods and Services per capita

Start with production function,

$$
\begin{aligned}
Y &= AK^{\alpha}L^{1-\alpha} \qquad \alpha \in [0,1] \\
\frac{Y}{L} &= AK^{\alpha}L^{1-\alpha} * \frac{1}{L} \\
\frac{Y}{L} &= A\left(\frac{K}{L}\right)^{\alpha}\left(\frac{L}{L}\right)^{1-\alpha} \\
y &= \underbrace{Ak^{\alpha}}_{f(k)}
\end{aligned}
$$

- $y = \frac{Y}{L}$: income per capita
- $k = \frac{K}{L}$: capital per capita

Diminishing Return to Capital



Demand of Goods and Services per capita

From GDP identity, demand comes from consumption and investment

$$Y = C + I$$

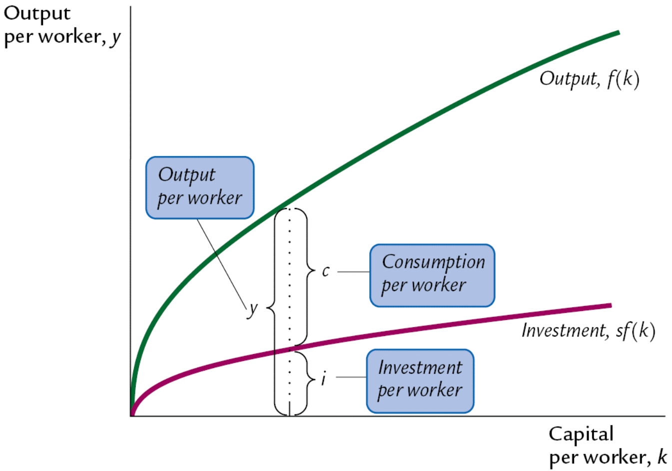
Can re-write GDP identity as per capita

$$\begin{aligned} Y &= C + I \\ \frac{Y}{L} &= \frac{C}{L} + \frac{I}{L} \\ y &= c + i \end{aligned}$$

Consumption Function

- s = saving rate, the fraction of income that is saved (s is an exogenous parameter), and consume rest of income $(1-s)$.
- It implies
  - Again, $y = f(k)$
  - $i = sy = sf(k)$
  - $c = y - i = y - sy = (1-s)y = (1-s)f(k)$

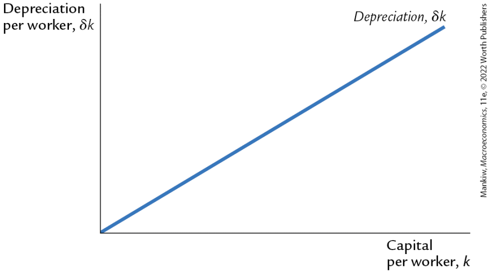
Output, Consumption, and Investment



Depreciation

Assume capital k depreciate at constant rate δ over time

- i.e. A million \$ machine depreciates at $10\%(\delta = 0.1)$ after each year.
- It implies δk capitals are depreciated after each period



Capital Accumulation

Basic idea: Investment increases the capital stock; depreciation reduces it.

$$\text{change in capital stock} = \text{investment} - \text{depreciation}$$
$$\Delta k = i - \delta k$$

Since $i = sf(k)$, then

$$\Delta k = sf(k) - \delta k$$

The Equation of Motion for k

$$\Delta k = sf(k) - \delta k$$

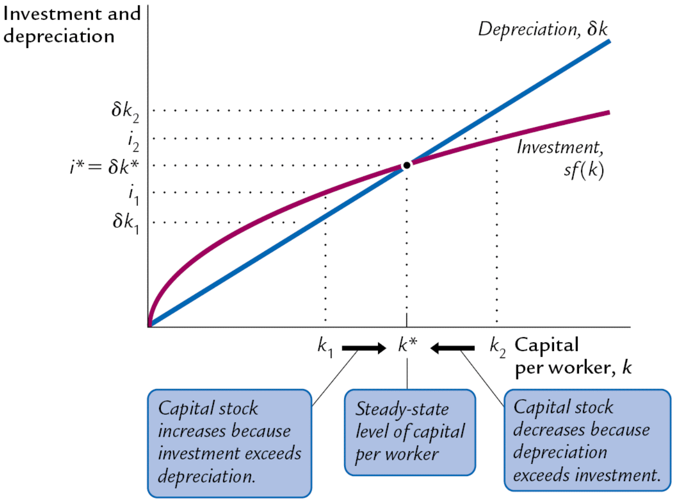
- The Solow model's central equation
- Determines behavior of capital over time . . .
- . . . which, in turn, determines behavior of all the other endogenous variables because they all depend on k.

Steady State

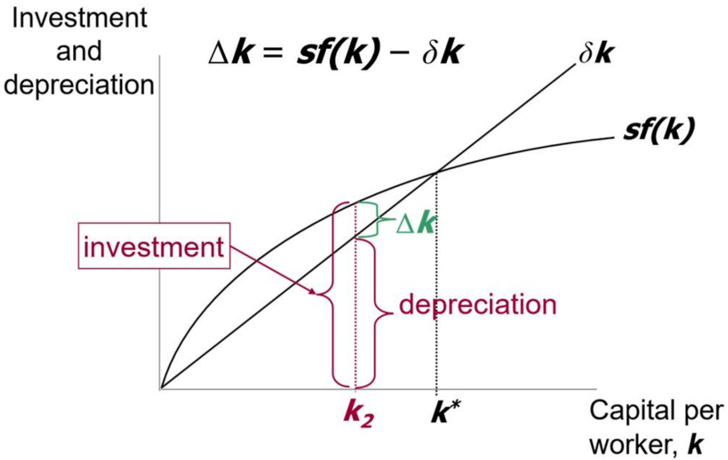
$$\Delta k = sf(k) - \delta k$$

- If investment is just enough to cover depreciation $\Rightarrow sf(k) = \delta k$
- $\Rightarrow$ capital per worker will remain constant: $\Delta k = 0$
- This occurs at one value of k, denoted k^*, called **steady state capital stock**

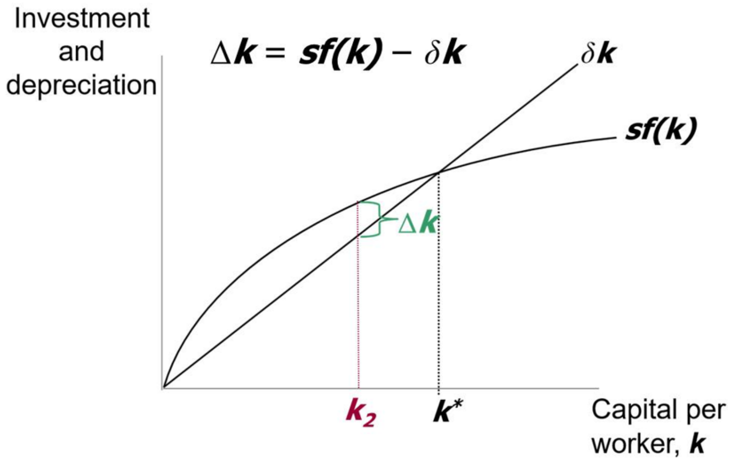
Steady State



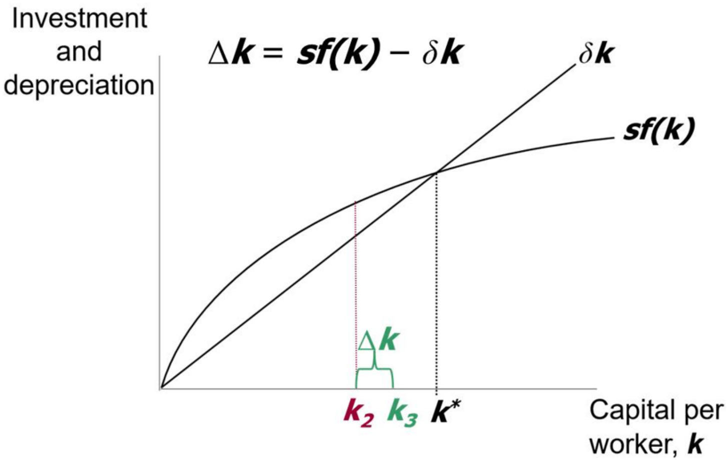
Moving Toward the Steady State



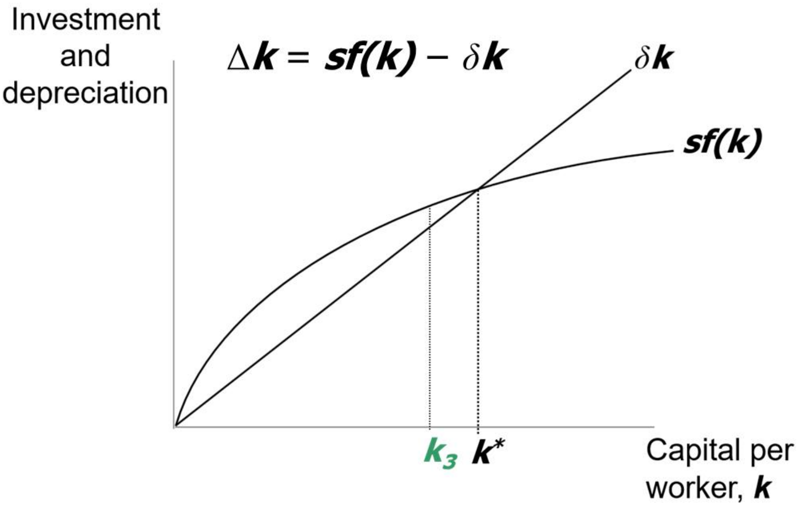
Moving Toward the Steady State



Moving Toward the Steady State



Moving Toward the Steady State



A Numerical Example

Suppose $Y = F(K, L) = AK^{0.5}L^{0.5} \Rightarrow y = Ak^{0.5}$, assume

- $s = 0.3$
- $\delta = 0.1$
- initial $k = 4$
- initial period? second period?

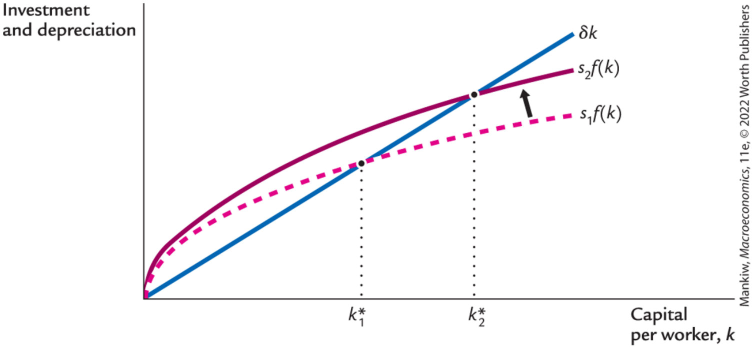
A Numerical Example

Year	k	y	c	i	δk	Δk
1	4.000	2.000	1.400	0.600	0.400	0.200
2	4.200	2.049	1.435	0.615	0.420	0.195
3	4.395	2.096	1.467	0.629	0.440	0.189
4	4.584	2.141	1.499	0.642	0.458	0.184
5	4.768	2.184	1.529	0.655	0.477	0.178
10	5.602	2.367	1.657	0.710	0.560	0.150
25	7.321	2.706	1.894	0.812	0.732	0.080
100	8.962	2.994	2.096	0.898	0.896	0.002
∞	9.000	3.000	2.100	0.900	0.900	0.000

How to Solve Steady State k^*?

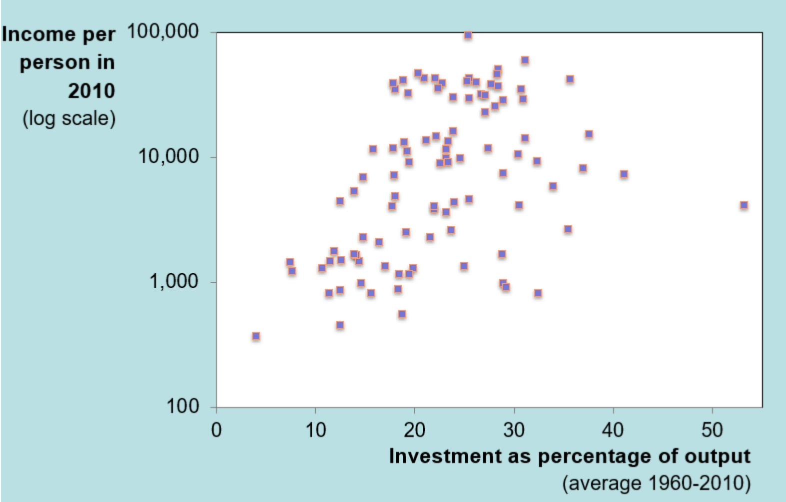
Filled by in-class notes

An Increasing in the saving rate



What will happen on k^* and y^*?

Evidence on investment rates and income per person



The Golden Rule: Introduction

- Different values of s lead to different steady states. How do we know which is the “best” steady state?
- The “best” steady state has the highest possible consumption per person c^*(we want consumption!)
- Thus, how can we find “right” s and k^* that maximize c^*
- This is called **Golden Rule**

Golden Rule

Start with steady state consumption c^*

$$c^* = (1 - s)f(k^*) = f(k^*) - i^*$$

At steady state, we know $i^* = \delta k^*$, thus

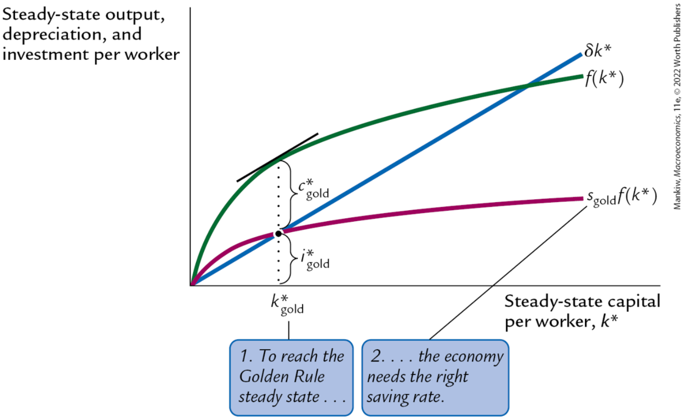
$$c^* = f(k^*) - \delta k^*$$

Since we want to maximize c^*, so take the derivative respect to k^* then set it equals to 0,

$$\frac{\partial c}{\partial k^*} = \frac{\partial f(k^*)}{\partial k^*} - \delta = 0$$

The last equation implies: $MPK = \delta$

The Golden Rule

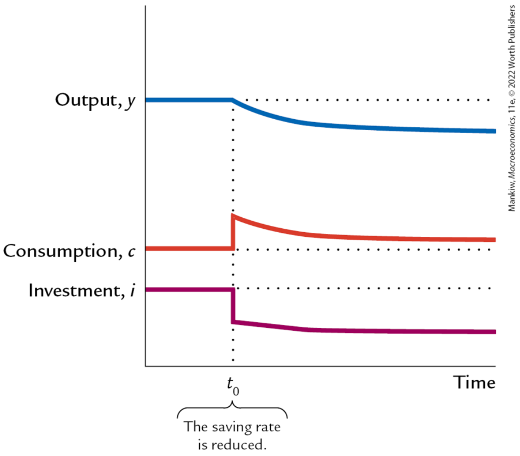


So, what if we increase s?

The transition to the Golden Rule steady state

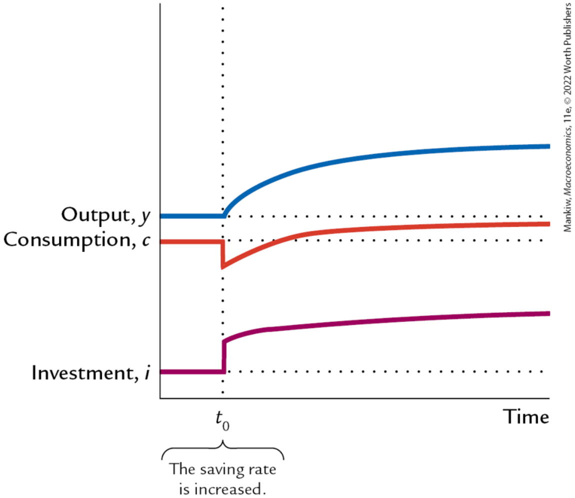
- The economy does not have a tendency to move toward the Golden Rule steady state.
- Achieving the Golden Rule requires that policymakers adjust s.
- This adjustment leads to a new steady state with higher consumption.
- But what happens to consumption during the transition to the Golden Rule?

Starting with too much capital



Implies $k^* > k^{gold} \Rightarrow$ then increasing c^* requires a fall in s.

Starting with too little capital



Solow Growth II
- Population Growth and Technological Progress

Population growth in Solow

In previous section, we assume there is no population growth, we relax assumption here

- Assume that population and labor force grow at rate n

$$\frac{\Delta L}{L} = n$$

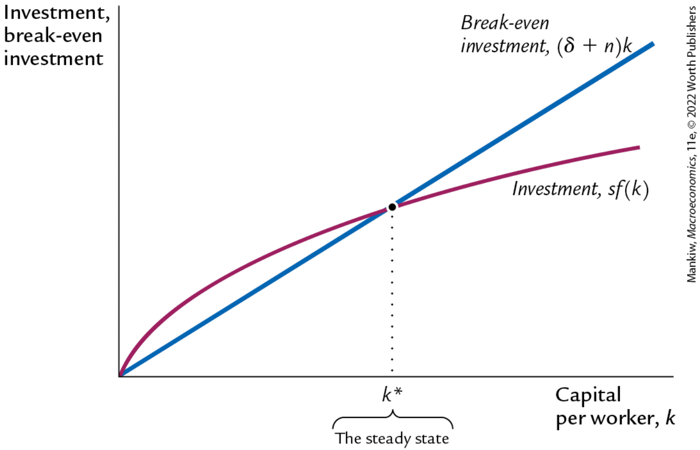
- Suppose $L = 1000$ in year 1 and the population is growing at 2% per year, then $\Delta L = nL = 0.02 * 1000 = 20$, so $L = 1020$ in year 2.

Break-even investment

- $(\delta + n)k$: break-even investment, the amount of investment necessary to keep k constant
- Break-even investment includes,
  - δk to replace capital as wears out
  - nk to equip new workers with capital (Otherwise, k would fall as the existing capital stock is spread more thinly over a larger population of workers.)
- the equation of motion for k with population growth

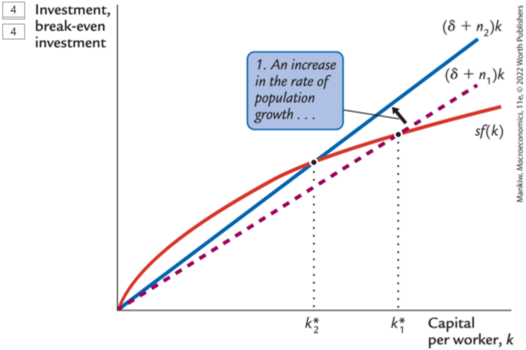
$$\Delta k = \underbrace{sf(k)}_{\text{actual investment}} - \underbrace{(\delta + n)k}_{\text{break-even investment}}$$

The Solow model diagram

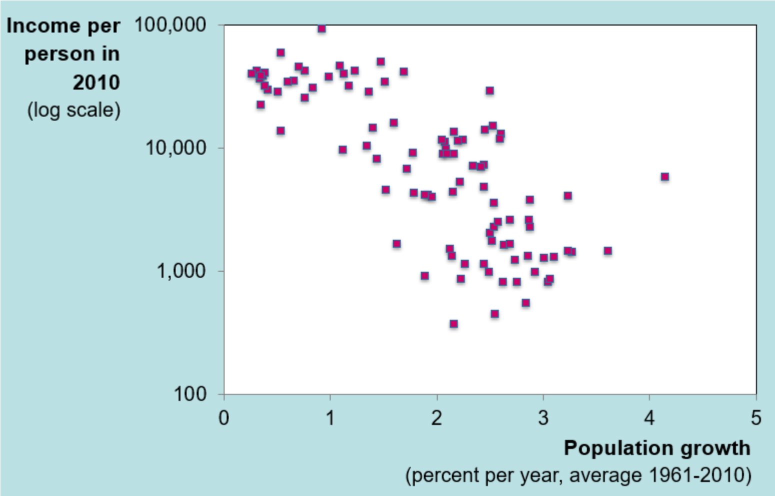


The impact of population growth

- An increase in ***n*** causes an increase in break-even investment, leading to a lower steady-state level of ***k***.



Evidence on population growth and income per person



The Golden Rule with population growth

Similar to previous section, express c^* in terms of k^*

$$
\begin{aligned}
c^* &= y^* - i^* \\
&= f(k^*) - (\delta + n)k^*
\end{aligned}
$$

- Choose k^* to maximize c^*

$$MPK = \delta + n$$

Alternative perspectives on population growth

The Malthusian Model (1798)

- It predicts population growth will outstrip the Earth's ability to produce food, leading to the impoverishment of humanity.
- Since the time of Malthus, world population has increased sixfold, yet living standards are higher than ever.
- Malthus neglected the effects of technological progress.

The Kremerian model (1993)

- Posits that population growth contributes to economic growth.
- More people = more geniuses, scientists, and engineers, so faster technological progress.
- Evidence from very long historical periods shows that:
  - as world population growth rate increased, so did the rate of growth in living standards.
  - historically, regions with larger populations have enjoyed faster growth.

Adding technological progress

In the simplest Solow model

- production technology is held constant
- income per capita is constant in the steady state
- in fact, U.S. real GDP per capita grew by a factor of over 4

A new variable: $E =$labor efficiency (labor productivity)

- Assume technological progress is labor-augmenting: it increases labor efficiency at the constant rate g

$$g = \frac{\Delta E}{E}$$

Adding technological progress

We now write production function as,

$$Y = F(K, L \times E)$$

- where $L \times E$ = number of effective workers
  - increases in labor efficiency have the same effect on output as increases in labor force.

Notation,

- $\hat{y} = \frac{Y}{LE}$: output per effective worker
- $\hat{k} = \frac{K}{LE}$: capital per effective worker
- $\hat{y} = f(\hat{k})$: production per effective worker
- $\hat{i} = sf(\hat{k})$: investment per effective worker

Break-even investment

Now, new break-even investment: $(\delta + n + g)k$,

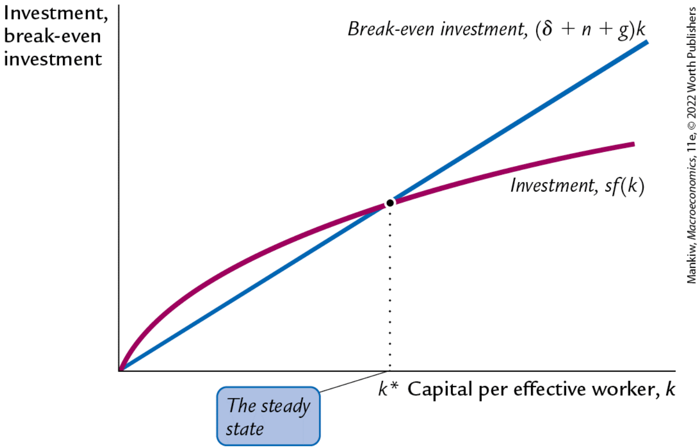
- amount of investment necessary to keep k constant
- gk: to provide capital for the new effective workers created by technological progress

Law of motion of capital per effective worker is ,

$$\Delta\hat{k} = sf(\hat{k}) - (\delta + n + g)\hat{k}$$

- notes: $\Delta\hat{k} = 0$ and $\Delta\hat{y} = 0$ at steady state

Technological progress diagram



Implication of technological change in the Solow model

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Golden Rule

Similar to previous section, express c^* in terms of k^*

$$\begin{aligned}\hat{c}^* &= \hat{y}^* - \hat{i}^* \\ &= f(\hat{k}^*) - (\delta + n + g)\hat{k}^*\end{aligned}$$

- Choose $\hat{k}^*$ to maximize $\hat{c}^*$

$$MPK = \delta + n + g$$

Example

Suppose we have Cobb-Douglas production function

$$Y = F(K, LE) = (K)^{0.5}(LE)^{0.5}$$

- assume $n = 0.02$, $\delta = 0.05$, $g = 0.03$
- derive $\hat{k}^*$ at steady state